

Table 4.1 Results for the eight algorithms

| K | n | [BFGS] | [PLS] | [PLS – BFGS] | [BEST ϕ] | [BFGS2] | [PLS2] | [PLS – BFGS2] | [BEST ϕ 2] |
|--------|-----|----------------|---------------|---------------|----------------|----------------|--------------|---------------|-----------------|
| 1 | 2 | 34/43/43 | 22/29/71 | 22/29/71 | 28/34/61 | 35/56/56 | 22/29/71 | 22/29/71 | 28/34/61 |
| 2 | 2 | 9/10/10 | 7/8/20 | 7/8/20 | 9/10/18 | 19/20/20 | 7/8/20 | 7/8/20 | 9/10/18 |
| 3 | 2 | 148/191/191 | 92/120/302 | 92/120/302 | 114/118/231 | 150/195/195 | 92/120/302 | 92/120/302 | 114/118/231 |
| 4 | 2 | 11/37/37 | 8/12/26 | 7/34/46 | 9/12/20 | 12/16/16 | 8/12/26 | 7/34/46 | 9/12/20 |
| 5 | 2 | 14/18/18 | 7/11/23 | 7/12/24 | 11/14/24 | 18/19/19 | 7/11/23 | 7/12/24 | 11/14/24 |
| 6 | 2 | F3/F3/F3 | 10/11/29 | 10/11/29 | 13/14/26 | 20/21/21 | 10/11/29 | 10/11/29 | 13/14/26 |
| 7 | 3 | 27/36/36 | 18/21/55 | 17/22/54 | 21/25/45 | 26/29/29 | 15/22/50 | 15/19/47 | 19/24/42 |
| 8 | 3 | 19/21/21 | 11/12/32 | 11/12/32 | 13/14/26 | 28/29/29 | 12/14/36 | 12/14/36 | 16/18/33 |
| 9 | 3 | 2/3/3 | 2/3/5 | 2/3/5 | 2/3/4 | 3/5/5 | 2/3/5 | 2/3/5 | 2/3/4 |
| 10 | 3 | 2/14/14* | F3/F3/F3 | 2/14/16* | 2/5/6* | 389/512/512 | F3/F3/F3 | 2/14/16* | 2/5/6* |
| 11 | 3 | 14/23/23 | 6/8/18 | 6/9/19 | 19/24/42 | 17/22/22 | 12/14/36 | 11/14/34 | 16/22/37 |
| 12 | 3 | 23/27/27 | 9/12/28 | 9/12/28 | 18/20/37 | 36/42/42 | 10/13/31 | 10/13/31 | 17/19/35 |
| 13 | 4 | 35/44/44 | 20/22/60 | 20/22/60 | 23/25/47 | 51/52/52 | 20/21/59 | 20/21/59 | 31/33/63 |
| 14 | 4 | 76/118/118 | 42/52/134 | 42/60/142 | 52/68/119 | 36/52/52 | 54/70/176 | F2/F2/F2 | 67/87/153 |
| 15 | 4 | 21/26/26 | 14/19/45 | 14/19/45 | 19/26/44 | 32/35/35 | 15/19/47 | 15/19/47 | 16/21/36 |
| 16 | 4 | 25/51/51 | 17/18/50 | 17/18/50 | 19/20/38 | 35/36/36 | 16/17/47 | 16/17/47 | 15/16/30 |
| 17 | 5 | F3/F3/F3 | 33/39/103 | F3/F3/F3 | 44/53/96 | 97/104/104 | 43/54/138 | F3/F3/F3 | 49/61/109 |
| 18 | 6 | 33/39/39 | 22/23/65 | 21/24/64 | 31/36/66 | 44/46/46 | 22/27/69 | 29/32/88 | 30/35/64 |
| 19 | 11 | 49/71/71 | 41/50/130 | 41/50/130 | 43/52/94 | 74/79/79 | 49/57/153 | 49/57/153 | 51/59/109 |
| 20 | 12 | 43/54/54 | 28/30/84 | 28/30/84 | 31/32/62 | 86/94/94 | 36/38/108 | 36/38/108 | 41/44/84 |
| 21 | 12 | 118/167/167 | 77/87/239 | 77/87/239 | 77/90/166 | 35/56/56 | 22/29/71 | 22/29/71 | 28/34/61 |
| 22 | 12 | 61/82/82 | 55/58/166 | 55/58/166 | 56/58/113 | 51/52/52 | 25/26/74 | 25/26/74 | 31/33/63 |
| 23 | 12 | 175/245/245 | 80/97/255 | 69/85/221 | 109/127/235 | 51/63/63 | 35/42/110 | 35/42/110 | 41/52/92 |
| 24 | 12 | 221/325/325 | 261/352/872 | 221/302/742 | 1286/1906/3191 | 143/179/179 | 103/147/351 | 102/145/347 | 118/155/272 |
| 25 | 12 | 23/24/24 | 15/16/44 | 15/16/44 | 21/22/42 | 21/22/22 | 15/16/44 | 15/16/44 | 21/22/42 |
| 26 | 12 | 29/37/37 | 23/31/75 | 23/31/75 | 25/34/58 | 27/33/33 | 23/27/71 | 23/28/72 | 24/28/51 |
| 27 | 12 | 11/12/12 | 8/9/23 | 8/9/23 | 6/7/12 | 12/16/16 | 8/9/23 | 8/9/23 | 6/7/12 |
| 28 | 12 | 17/29/29 | 13/15/39 | 13/15/39 | 13/15/27 | 34/36/36 | 14/16/42 | 14/16/42 | 15/17/31 |
| 29 | 12 | 8/11/11 | 5/6/14 | 5/6/14 | 6/7/12 | 7/8/8 | 4/5/11 | 4/5/11 | 5/6/10 |
| 30 | 12 | 22/48/48 | 18/19/53 | 18/19/53 | 19/20/38 | 22/23/23 | 16/17/47 | 16/17/47 | 17/18/34 |
| 31 | 12 | 50/113/113 | 25/26/74 | 25/26/74 | 26/27/52 | 39/40/40 | 20/21/59 | 20/21/59 | 21/22/42 |
| TOTALS | | 1242/1787/1787 | 904/1114/2868 | 850/1078/2724 | 2054/2778/4805 | 1108/1303/1303 | 630/780/1986 | 634/804/2018 | 752/876/1601 |

* algorithm terminated at a point which is not the minimum.

Table 4.2 Further results for the eight algorithms

| K | n | [BFGS] | [PLS] | [PLS – BFGS] | [BEST ϕ] | [BFGS2] | [PLS2] | [PLS – BFGS2] | [BEST ϕ 2] |
|--------|-----|----------------|--------------|--------------|----------------|-------------|--------------|---------------|-----------------|
| 20 | 100 | 70/95/95 | 40/43/121 | 40/43/121 | 42/44/85 | 198/206/206 | 68/71/205 | 68/71/205 | 72/76/147 |
| 21 | 100 | 376/781/781 | 432/446/1308 | 432/446/1308 | 695/719/1413 | 35/56/56 | 22/29/71 | 22/29/71 | 28/34/61 |
| 22 | 100 | 163/261/261 | 128/130/384 | 128/130/384 | 161/163/323 | 55/56/56 | 25/26/74 | 25/26/74 | 33/35/67 |
| 23 | 100 | 256/356/356 | 78/87/241 | 78/87/241 | 170/194/363 | 68/78/78 | 37/42/114 | 37/42/114 | 50/52/101 |
| 24 | 100 | F3/F3/F3 | F2/F2/F2 | F2/F2/F2 | F2/F2/F2 | F2/F2/F2 | F2/F2/F2 | F2/F2/F2 | F2/F2/F2 |
| 25 | 100 | 350/436/436 | F2/F2/F2 | F2/F2/F2 | 52/53/104 | 36/37/37 | 25/26/74 | 27/28/80 | 36/37/72 |
| 26 | 100 | 42/46/46 | 36/43/113 | 36/43/113 | 38/43/80 | 39/42/42 | 36/41/111 | 36/41/111 | 37/44/80 |
| 27 | 100 | 3/4/4 | 3/4/8 | 3/4/8 | 3/4/6 | 3/4/4 | 3/4/8 | 3/4/8 | 3/4/6 |
| 28 | 100 | 126/203/203 | 96/98/288 | 96/98/288 | 96/98/193 | 215/217/217 | 100/102/300 | 100/102/300 | 100/102/201 |
| 29 | 100 | 8/11/11 | 5/6/14 | 5/6/14 | 6/7/12 | 7/8/8 | 5/6/14 | 5/6/14 | 6/7/12 |
| 30 | 100 | 106/237/237 | 53/54/158 | 53/54/158 | 54/55/108 | 21/22/22 | 22/23/65 | 22/23/65 | 21/22/42 |
| 31 | 100 | 153/437/437 | 45/46/134 | 45/46/134 | 50/51/100 | 43/44/44 | 22/23/65 | 22/23/65 | 23/24/46 |
| TOTALS | | 1303/2431/2431 | 916/957/2769 | 916/957/2769 | 1315/1378/2683 | 684/733/733 | 340/367/1027 | 340/367/1027 | 373/400/763 |

Table 4.3 Results for the eight algorithms with $10x^{(1)}$ as starting point

| K | n | [BFGS] | [PLS] | [PLS – BFGS] | [BEST ϕ] | [BFGS2] | [PLS2] | [PLS – BFGS2] | [BEST ϕ 2] |
|--------|-----|----------------|--------------|--------------|----------------|-------------|--------------|---------------|-----------------|
| 20 | 12 | 43/54/54 | 28/30/84 | 28/30/84 | 31/32/62 | 86/94/94 | 36/38/108 | 36/38/108 | 41/44/84 |
| 21 | 12 | 315/419/419 | 207/254/666 | 207/254/666 | 262/324/585 | 57/75/75 | 81/106/266 | 81/106/266 | 84/93/176 |
| 22 | 12 | 123/170/170 | 64/68/194 | 64/68/194 | 89/90/178 | 63/73/73 | 40/46/124 | 40/46/124 | 48/51/98 |
| 23 | 12 | 258/367/367 | 72/80/222 | 71/83/223 | 154/179/332 | 67/77/77 | 39/46/122 | 39/46/122 | 53/55/107 |
| 24 | 12 | F3/F3/F3 | 413/546/1370 | 456/606/1516 | 1001/1338/2338 | 654/812/812 | 470/644/1582 | 470/640/1578 | 409/529/937 |
| 25 | 12 | 27/29/29 | 18/19/53 | 18/19/53 | 25/26/50 | 47/49/49 | 18/19/53 | 18/19/53 | 25/26/50 |
| 26 | 12 | 132/166/166 | 35/39/107 | 35/39/107 | 48/51/98 | 45/50/50 | 59/62/178 | 59/66/182 | 35/43/77 |
| 27 | 12 | F3/F3/F3 | F2/F2/F2 | F3/F3/F3 | 908/1406/2313 | 54/55/55 | 41/44/124 | F3/F3/F3 | 32/38/69 |
| 28 | 12 | 19/30/30 | 16/18/48 | 16/18/48 | 17/19/35 | 36/38/38 | 18/20/54 | 18/20/54 | 18/20/37 |
| 29 | 12 | 14/15/15 | 8/9/23 | 8/9/23 | 9/10/18 | 9/10/10 | 7/8/20 | 7/8/20 | 8/9/16 |
| 30 | 12 | 116/193/193 | 29/30/86 | 29/30/86 | 34/35/68 | 89/90/90 | 29/30/86 | 29/30/86 | 34/35/68 |
| 31 | 12 | 196/358/358 | 63/66/190 | 63/66/190 | 65/66/130 | 146/147/147 | 40/41/119 | 40/41/119 | 47/48/94 |
| TOTALS | | 1243/1801/1801 | 540/613/1673 | 539/616/1674 | 734/832/1556 | 645/703/703 | 367/416/1130 | 367/420/1134 | 393/424/807 |